

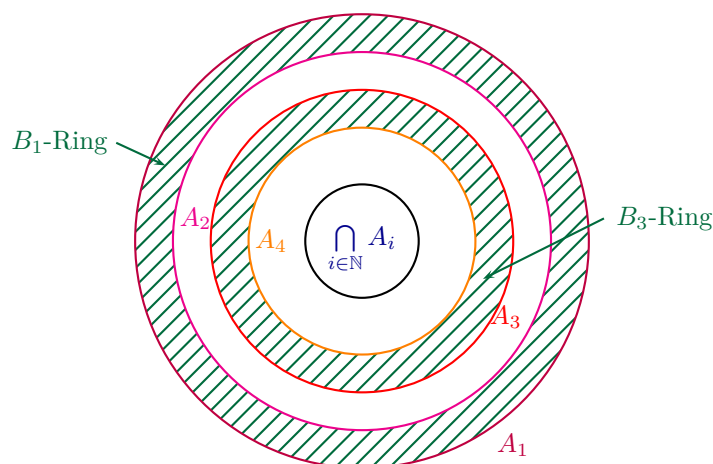
$$\begin{aligned}\mu\left(\sum_{i=1}^{\infty} B_i\right) &= \mu\left(\bigcup_{n \in \mathbb{N}} A_n\right) \stackrel{\text{Vor.}}{=} \lim_{n \rightarrow \infty} \mu(A_n) \\ &= \lim_{n \rightarrow \infty} \mu\left(\sum_{i=1}^n B_i\right) \stackrel{\substack{\uparrow \\ \mu \text{ ist Inhalt}}}{=} \lim_{n \rightarrow \infty} \sum_{i=1}^n \mu(B_i) = \sum_{i=1}^{\infty} \mu(B_i)\end{aligned}$$

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a) $\Rightarrow$ c): ($\Leftrightarrow$ b) $\Rightarrow$ c))

Seien $A_1, A_2, \dots \in \mathcal{R}$ antiton mit $\bigcap_{i \in \mathbb{N}} A_i \in \mathcal{R}$ und $\mu(A_1) < \infty$.

Setze $B_n := A_n \setminus A_{n+1} \in \mathcal{R}$.



Dann gilt:

$$\begin{aligned}B_1, B_2, \dots \text{ p.d.} \quad \underline{\text{und}} \quad \sum_{i=1}^n B_i &= A_1 \setminus A_2 \cup A_2 \setminus A_3 \cup \dots \cup A_n \setminus A_{n+1} \\ &= A_1 \setminus A_{n+1} = A_1 \setminus \bigcap_{i=1}^{n+1} A_i\end{aligned}$$

$$\Rightarrow \sum_{i=1}^{\infty} B_i = A_1 \setminus \bigcap_{i=1}^{\infty} A_i$$

Begr.: „ $\subseteq$ “ klar

$$\begin{aligned}\text{„}\supseteq\text{“ Sei } \omega \in A_1 \setminus \bigcap_{i=1}^{\infty} A_i &\Rightarrow \omega \in A_1 \cap \left(\bigcup_{i=1}^{\infty} A_i^C\right) \\ &\Rightarrow \exists n : \omega \in A_1 \wedge \omega \in A_n^C \\ &\Rightarrow \exists n : \omega \in A_1 \cap A_n^C = A_1 \setminus A_n \Rightarrow \omega \in \sum_{i=1}^{n-1} B_i\end{aligned}$$

$$\Rightarrow \mu\left(\sum_{i=1}^{\infty} B_i\right) = \mu\left(A_1 \setminus \bigcap_{i=1}^{\infty} A_i\right) \stackrel{\substack{\uparrow \\ \mu(A_1) < \infty}}{=} \mu(A_1) - \mu\left(\bigcap_{i=1}^{\infty} A_i\right)$$

Es folgt:

$$\begin{aligned}\mu\left(\bigcap_{i=1}^{\infty} A_i\right) &= \mu(A_1) - \mu\left(\sum_{i=1}^{\infty} B_i\right) \\ &\stackrel{\substack{\uparrow \\ \mu \text{ ist } \text{Pr\"{a}ma\ss}}}{=} \mu(A_1) - \sum_{i=1}^{\infty} \mu(B_i) \\ &= \mu(A_1) - \sum_{i=1}^{\infty} (\mu(A_i) - \mu(A_{i+1})) \\ &\stackrel{\substack{\uparrow \\ \text{Teleskopsumme}}}{=} \mu(A_1) - \mu(A_1) + \lim_{n \rightarrow \infty} \mu(A_n)\end{aligned}$$

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c) $\Leftrightarrow$ d):

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